

INSTANTONS AND GLUEBALLS

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Abstract

We investigate the impact of instantons on scalar glueball properties in a largely model-independent analytical approach based on the instanton-improved operator product expansion (IOPE) of the 0^{++} glueball correlator. The instanton contributions turn out to be dominant, to substantially improve the consistency of the corresponding QCD sum rules, and to increase the glueball residue about fivefold.

1 Introduction

As the hadrons with the largest gluon content (despite probably sizeable quarkonium admixtures), glueballs provide a privileged source of information on how glue manifests itself in hadronic bound states (in their rest frame). Progress in understanding glueball structure may therefore even help to unravel the elusive gluonic component of the classical hadrons.

The intricacies of hadronic glue are exemplified by the problems encountered when implementing various effective, gluonic degrees of freedom, such as constituent gluons, strings, or flux tubes, into hadron models. The resulting effects are generally less prominent and more ambiguous than those of the quark substructure, for which phenomenologically rather successful effective degrees of freedom are provided, e.g., by constituent quarks. The problems in modeling the gluon sector are partially related to the fact that gluons, unlike quarks, do not carry internal, global quantum numbers (like flavor or charge) to which external probes can couple. Unambiguous and transparent gluonic effects in hadrons are therefore much harder to identify.

Model-independent and more directly QCD-based approaches are thus called upon to clarify the structure of hadronic glue and to identify potentially dominant gluon field configurations. Promising candidates for the latter, especially in the 0^{++} glueball channel, are instantons [1] (i.e. the strong, coherent gluon fields which, by mediating tunneling processes, give rise to the θ -angle of the QCD vacuum), as qualitative arguments [2, 3, 4] and instanton-liquid model results [5] suggest. In the following, we will give a brief summary of work

in an analytical approach to the scalar glueball correlator at short distances [4, 6], based on an instanton-improved operator product expansion (IOPE) and QCD sum-rule techniques, in which these issues can be addressed, and glueball properties calculated, in a largely model-independent fashion.

2 IOPE sum rules

Consider the correlation function

$$\Pi(-q^2) = i \int d^4x e^{iqx} \langle 0 | T O_S(x) O_S(0) | 0 \rangle \quad (1)$$

where the interpolating field

$$O_S = \alpha_s G_{\mu\nu}^a G^{a,\mu\nu} \quad (2)$$

carries the quantum numbers of the scalar (0^{++}) glueball. The standard OPE separates this correlator into contributions from hard field modes (with typical momenta $k \geq \mu \sim 1/2$ GeV), contained in perturbative Wilson coefficients, and from soft ($k < \mu$) modes contained in the vacuum expectation values of composite QCD operators, the so-called condensates. The perturbative Wilson coefficients of the low-dimensional operators can be found to $O(\alpha_s)$ in [7, 8].

Remarkably, the nonperturbative power (i.e. condensates) corrections to this conventional OPE turn out to be small and are, except for the lowest-dimensional gluon condensate contribution, negligible. The bulk of the non-perturbative physics responsible for the strong binding in the scalar glueball channel should therefore manifest itself in the Wilson coefficients, i.e. predominantly via direct instantons [2, 3] which are neglected in the standard analyses [7, 8]. This expectation is strengthened by the fact that instantons couple particularly strongly to the gluonic 0^{++} interpolators.

In Ref. [4, 6], we have evaluated the direct-instanton contributions and implemented them into the corresponding IOPE sum rules, which are based on the Borel-transformed moments

$$\mathcal{R}_k(\tau) = \hat{B} \left[(-Q^2)^k \Pi(Q^2) \right] \quad (3)$$

(for the explicit form of the Borel operator \hat{B} see, e.g. [7]) with $k \in \{-1, 0, 1, 2\}$. Besides the standard OPE contributions $\mathcal{R}_k^{(OPE)}$ of Ref. [7], the IOPE includes

the instanton contributions [4]

$$\mathcal{R}_k^{(I+\bar{I})}(\tau) = -2^6 \pi^2 \bar{n} \left(\frac{-\partial}{\partial \tau} \right)^{k+1} \times \left\{ \xi^2 e^{-\xi} \left[(1 + \xi) K_0(\xi) + \left(2 + \xi + \frac{2}{\xi} \right) K_1(\xi) \right] - 2 \right\}, \quad (4)$$

($k \geq -1$, $\xi \equiv \bar{\rho}^2 / (2\tau)$). The semiclassical result (4) neglects $O(\hbar)$ corrections (which are suppressed by the large instanton action $S_I(\bar{\rho}) \sim 10\hbar$) and multi-instanton correlations (since $|x| \sim |Q^{-1}| \leq 0.2$ fm is small compared with the average instanton separation $\bar{R} \sim 1$ fm). We have also removed the soft subtraction term $-\Pi^{(I+\bar{I})}(0) = -2^7 \pi^2 \bar{n}$ to avoid double-counting with the condensates. Below, the average instanton size $\bar{\rho} \simeq (1/3)$ fm and density $\bar{n} \simeq (1/2)$ fm⁻⁴ will be fixed at the values obtained (approximately) from instanton vacuum model [1] and lattice [9] simulations.

To obtain sum rules, the IOPE expressions are matched to their “phenomenological” counterparts, which are derived from the borelized dispersive representation

$$\mathcal{R}_k^{(ph)}(\tau) = \frac{1}{\pi} \int_0^\infty ds s^k \text{Im} \Pi^{(ph)}(s) e^{-s\tau} + \delta_{k,-1} \Pi^{(ph)}(0) \quad (5)$$

where the spectral function $\text{Im} \Pi^{(ph)}(s)$ contains a glueball pole contribution and an effective continuum from the dispersive cut of the IOPE, starting at an effective threshold s_0 . This corresponds to

$$\text{Im} \Pi^{(ph)}(s) = \pi f_G^2 m_G^4 \delta(s - m_G^2) + \text{Im} \left[\Pi^{(OPE)} + \Pi^{(I+\bar{I})} \right](s) \theta(s - s_0). \quad (6)$$

The instanton continuum contributions

$$\text{Im} \Pi^{(I+\bar{I})}(s) = -2^4 \pi^4 \bar{n} \bar{\rho}^4 s^2 J_2(\sqrt{s} \bar{\rho}) Y_2(\sqrt{s} \bar{\rho}) \quad (7)$$

(J_2 (Y_2) are Bessel (Neumann) functions), introduced in [4], will play an essential role in the subsequent analysis. The IOPE sum rules can then be written as

$$\mathcal{R}_k^{(IOPE)}(\tau) \equiv \mathcal{R}_k^{(I+\bar{I})}(\tau) + \mathcal{R}_k^{(OPE)}(\tau) = \mathcal{R}_k^{(ph)}(\tau, s_0). \quad (8)$$

The subtraction constant $\Pi^{(ph)}(0)$ in the \mathcal{R}_{-1} sum rule (regularized by removing the high-momentum contributions) is related to the gluon condensate by the low-energy theorem [2]

$$\Pi(0) = \frac{32\pi}{b_0} \langle \alpha G^2 \rangle. \quad (9)$$

This relation provides an important consistency check for the sum-rule analysis, as discussed below.

3 Results and conclusions

The quantitative analysis of the sum rules (8) amounts to determining those values of the glueball parameters and s_0 in Eq. (6) for which both sides optimally match in the fiducial τ domain (determined such that the approximations on both sides of the sum rules are expected to be reliable [4]). The previously neglected instanton contributions turn out to be dominant and render (8) the first overall consistent set of QCD sum rules in the scalar glueball channel.

In particular, the IOPE resolves two long-standing flaws of the earlier sum rules [2, 7, 8]: the mutual inconsistency between different Borel moments and the inconsistency with the low-energy theorem (9). Even the previously deficient and usually discarded lowest-moment (\mathcal{R}_{-1}) sum rule is rendered consistent both with the higher-moment sum rules and the low-energy theorem. (A subsequent analysis of the related gaussian variant of these sum rules can be found in Ref. [10].) No evidence for a low-lying ($m \ll 1$ GeV) gluonium state (or any state strongly coupled to gluonic interpolators), which had been argued for on the basis of this sum rule [2], remains.

All four IOPE sum rules show an unprecedented degree of consistency and allow for a simultaneous 3-parameter fit to the glueball mass, its coupling, and the continuum threshold s_0 . In Fig. 1 the glueball pole contribution (solid line) to the optimized \mathcal{R}_0 sum rule is compared with the remaining components (OPE with subtracted OPE-continuum (dash-double-dotted), instanton contribution (dashed), instanton continuum (dash-dotted), and their sum (dotted)). Both parts match almost perfectly over the whole fiducial region, mostly due to the dominant instanton contributions. The remaining sum rules (including the \mathcal{R}_{-1} sum rule) show a similarly high degree of stability. Together with the mutual agreement of all four IOPE sum rules (in the typical range of uncertainty) and their consistency with the low-energy theorem, this indicates that the IOPE provides a sufficiently complete description of the short-distance glueball correlator. The most dramatic quantitative effect of the direct instantons is to increase the residuum of the glueball pole, f_G^2 , by about a factor of five. From the \mathcal{R}_2 sum rule (likely to be the most reliable one since it receives the strongest relative pole contribution) we obtain $m_G = 1.53 \pm 0.2$ GeV and $f_G = 1.01 \pm 0.25$ GeV.

The instanton continuum contributions, Eq. (7), are indispensable for the overall consistency and stability of the sum rules and shed new light on the

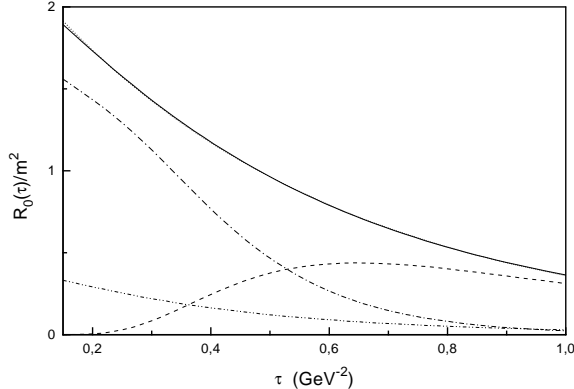


Figure 1: Contributions to the \mathcal{R}_0 sum rule, as explained in the text.

spectral content of the glueball correlator. For once, they compensate, together with the perturbative terms, the pole contribution and thereby lead to an improved description of the correlator towards low momenta. This reconciles the sum rules with the low-energy theorem in the $Q \rightarrow 0$ limit. In the opposite limit, i.e. for $\tau \rightarrow 0$, the instanton part of the continuum remains effective despite the suppression incurred by funnelling a sizeable momentum through a coherent field. This indicates that small-instanton physics accounts for part of the higher-lying strength in (1) and may play a rather prominent role in excited glueball states.

In contrast to previously studied IOPE sum rules [11], built on quark-based correlators, those for the scalar glueball are the first where i) the instanton contributions do not enter via topological quark zero-modes and where ii) the sum rules reach a satisfactory (though not excellent) level of consistency even without any perturbative and soft contributions, i.e. with the instanton terms alone. The latter result explains why instanton models find scalar glueball properties similar to those obtained above [5]. It also illustrates that the quantitative impact of the instanton contributions can be judged only if all contributions are consistently taken into account. At present, the IOPE seems to be the only controlled and analytical framework in which this is possible.

The predominance and approximate self-sufficiency of the instanton contributions indicates that instantons may generate the bulk of the forces which bind the scalar glueball. A further, striking consequence of the exceptionally strong instanton contributions is that the scales of the predicted 0^{++} glueball properties are approximately set by the bulk features of the instanton size distribution. More specifically, neglecting the standard OPE contributions, one

finds

$$m_G \sim \bar{\rho}^{-1}, \quad (10)$$

$$f_G^2 \sim \bar{n}\bar{\rho}^2. \quad (11)$$

Conceptually, the main virtue of these scaling relations lies in establishing an explicit link between fundamental vacuum and hadron properties. They could also be of practical use, e.g. for the test of instanton vacuum models, to provide constraints for glueball model building, or generalized to finite temperature and baryon density, where the instanton distribution changes. If the glueball size r_G scales like its Compton wavelength, one furthermore has $r_G \sim \bar{\rho}$, in agreement with the lattice calculations [12] which find $r_G \sim 0.2$ fm.

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